Sparse Recovery of Strong Reflectors With an Application to Non-Destructive Evaluation

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IEEE International Ultrasonics Symposium, October 25, 2018

Outline

Motivation

Measurement setup

Problem statement

Proposed approach

- TOF recovery
- TOF matching
- Localization

Results

Conclusion

Motivation

How can we reduce data rate / increase frame rate?

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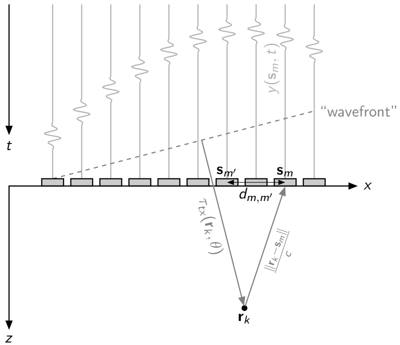
To what extent can recent signal processing techniques, e.g.:

1. *Finite rate of innovation (FRI)*
2. *Euclidean distance matrices (EDMs)*

*be applied to ultrasound (US) imaging / **localization**?*

Measurement setup: plane wave insonification



Time-of-flight (TOF) for element at $\mathbf{s}_m$ and point at $\mathbf{r}_k = [x_k, z_k]^T$:

$$\tau(\mathbf{r}_k, \mathbf{s}_m, \theta) = (x_k \sin\theta + z_k \cos\theta)/c + \|\mathbf{r}_k - \mathbf{s}_m\|/c.$$

Objective

Problem statement

Given discrete measurements $\{y_m[n]\}_{n=0}^{N-1}$ *at the element positions* $\{\mathbf{s}_m\}_{m=0}^{M-1}$*, estimate the locations of the reflectors* $\{\mathbf{r}_k\}_{k=0}^{K-1}$.

[1] C. Prada and J.L. Thomas,"Experimental subwavelength localization of scatterers by decomposition of the time reversal operator interpreted as a covariance matrix," 2003.

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- Exploit minimum *degrees of freedom* (DOF): least amount of elements M and samples per element N.
- Continuous recovery.

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Existing approaches discretize medium:

- Sparse deconvolution.
- Subspace approach $\Rightarrow K \leq M$.[1]

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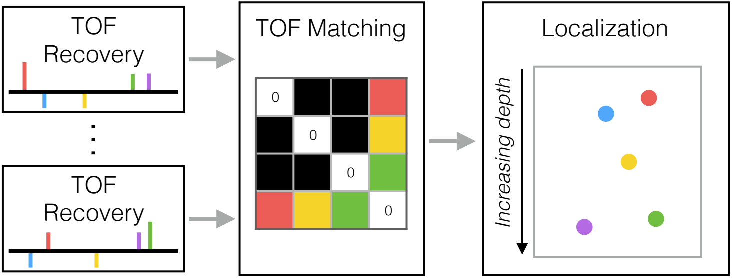
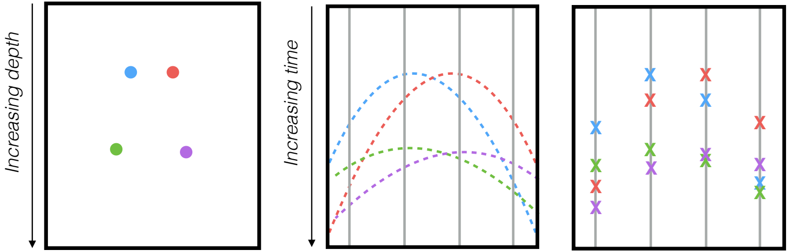
Proposed approach



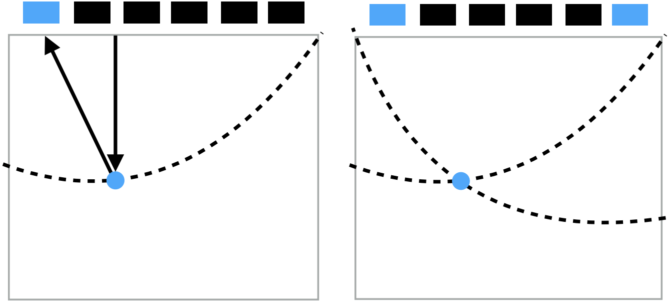
Simple example



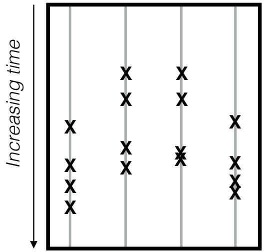
(a) Image domain. (b) Measurement domain. (c) Labeled TOFs

Localization with labeled TOFs

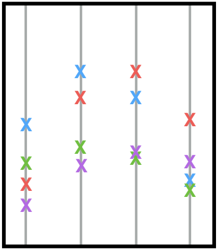
Recover reflector position from TOFs of at least two elements.



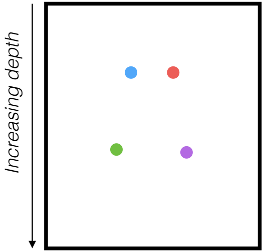
Proposed approach applied to simple example



(a) Recovered TOFs.



(b) TOF matching.



(c) Localization.

TOF recovery

For K reflectors, we receive the following *pulse stream* at $\mathbf{s}_m$:

$$y_m(t) = \sum_{k=0}^{K-1} \underbrace{\frac{a_k}{2\pi \|\mathbf{r}_k - \mathbf{s}_m\|}}_{a_{m,k}} h\Big(t - \underbrace{\tau(\mathbf{r}_k, \mathbf{s}_m)}_{\tau_{m,k}}\Big),$$

where $h(t)$ is known pulse shape.

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- $2K$ DOF: $\{a_{m,k}\}_{k=0}^{K-1}$ and $\{\tau_{m,k}\}_{k=0}^{K-1}$.

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- $2K$ DOF: $\{a_{m,k}\}_{k=0}^{K-1}$ and $\{\tau_{m,k}\}_{k=0}^{K-1}$.
- *Finite rate of innovation* (FRI) sampling and recovery.[2]
- At least $N \geq 2K+1$ samples.

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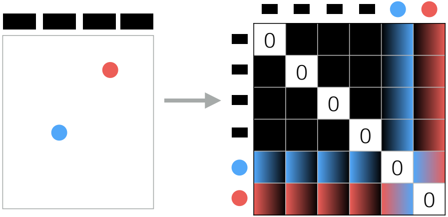
And Now For Something
Completely Different

An overview of *Euclidean distance matrices* (EDMs)

- Consider P points (elements and reflectors) $\{\mathbf{x}_p\}_{p=0}^{P-1}$ in a D-dimensional Euclidean space.
- For US, $P = M + K$ and $D = 2$.



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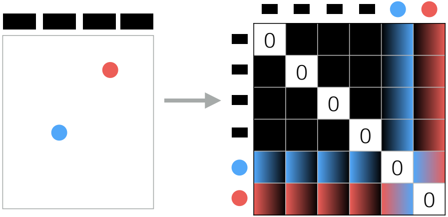

- Entry at i-th row and j-th column of an EDM $\mathbf{E} \in \mathbb{R}^{(P \times P)}$:

$$\mathbf{E}_{(i,j)} = \|\mathbf{x}_i - \mathbf{x}_j\|^2 = \mathbf{x}_i^T \mathbf{x}_i - 2\mathbf{x}_i^T \mathbf{x}_j + \mathbf{x}_j^T \mathbf{x}_j.$$

An overview of EDMs (cont.)

- Matrix formulation with $\mathbf{X} = [\mathbf{x}_0, \ldots, \mathbf{x}_{P-1}]$:

$$\mathbf{E} = \mathbf{1}\text{diag}(\mathbf{X}^T\mathbf{X})^T - 2\mathbf{X}^T\mathbf{X} + \text{diag}(\mathbf{X}^T\mathbf{X})\mathbf{1}^T.$$

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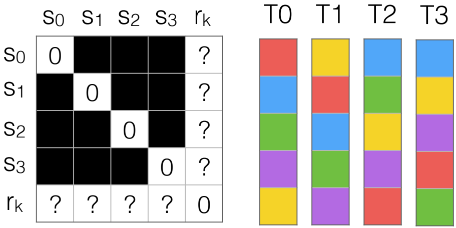
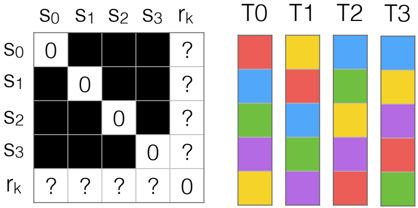
- For $P \geq D \Rightarrow \text{rank}(\mathbf{X}^T\mathbf{X}) \leq D \Rightarrow \text{rank}(\mathbf{E}) \leq D + 2$.
- Assuming perfect TOF recovery, we have entries of EDM but need to determine their position.



TOF matching

1. With $D + 2$ elements form an EDM with maximum rank.
2. Augment EDM with different combos of recovered TOFs.[3]
   - Estimate / remove transmit time and multiply with c.
   - Incorrect combos will increase rank.
   - Correct ones will not!



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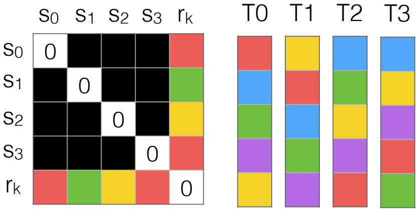

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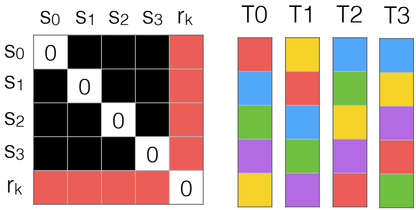

K TOFs, M channels $\Rightarrow K^M$ combinations.

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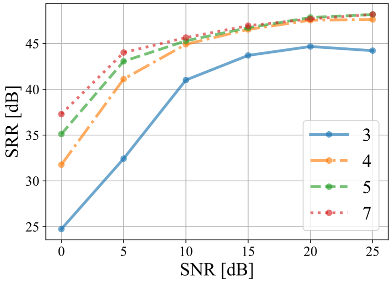
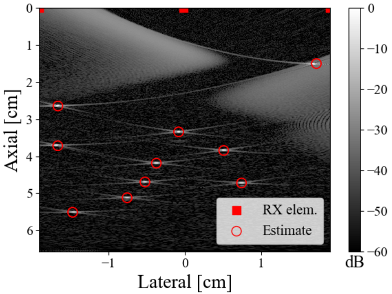
Simulation with Field II





- 50 unique configurations of 10 reflectors at varying SNR.
- 128 transmit elements, single-cycle square wave excitation, and Gaussian-modulated sinusoidal impulse response ($f_c = 5.208\,\text{MHz}$ and $bw = 67\,\%$).

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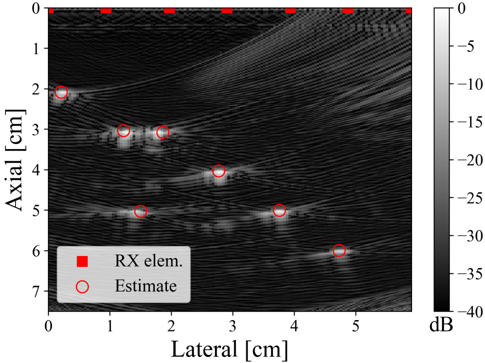
In-vitro non-destructive evaluation scenario



- Aluminum block with drilled holes.
- 64 transmit elements, $f_c = 5\,\text{MHz}$.

Conclusion

- Exploit minimum DOF for localization.
  - FRI along each element $\Rightarrow N \geq 2K + 1$ samples.
  - EDM across elements $\Rightarrow M \geq 3$ elements (using Gram matrix).

[4] More detail in: E. Bezzam, "Sampling at the rate of innovation of ultrasound imaging and localization," 2018.

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- In the paper:[4]
  - More on TOF recovery.
  - Dealing with noise for TOF recovery and matching.
  - Easing the combinatorial matching problem.

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THANK YOU FOR YOUR ATTENTION!

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